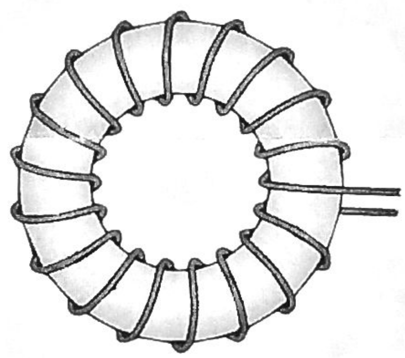
Electricity and Magnetism, Test 3, 13/04/2022

4 questions, 50 points

Write your name and student number on each answer sheet. Use of a calculator is allowed. You may make use of your own notes, the lecture notes, the book (hardcopy or digital) and the provided formula sheet. You are not allowed to have an active internet connection on your tablet or laptop. The same notation is used as in the book, i.e. a bold-face **A** is a vector, $\hat{\boldsymbol{x}}$ is the unit vector in the x-direction, and T is a scalar.
In your handwritten answers, remember to indicate vectors (unit vectors) with an arrow (hat) above the symbol.

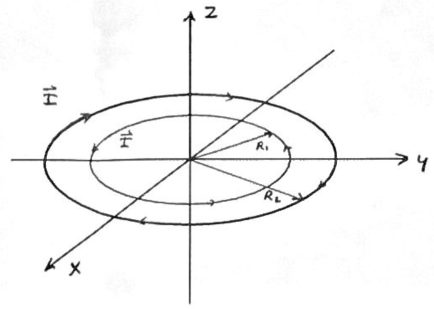
1. A toroidal coil
   A toroidal coil consists of a circular ring, around which a long wire is wrapped. The basic shape (with a low number of windings) is shown in the figure on the right. We consider a coil whose winding is uniform and tight enough so that each turn can be considered a plane closed loop. The toroid ring is made of a material with a linear susceptibility χ_m. A coil makes N turns around the ring, with a current I going through the coil. For this question, you can neglect the effect of the current leads that are seen on the right of the figure.



   (a) (5 points) Find the general expression for **B** inside the central plane of the toroid ring.
   (b) (5 points) Find the magnetic field outside the toroidal coil.
   (c) (5 points) Suppose the toroid material is diamagnetic. Sketch the direction of the magnetisation of this material and the direction of the surface bound current. In your sketch also include the direction of the current in the coil.
   (d) (5 points) Suppose we use instead a toroid material that is ferromagnetic. We magnetize this material by running a large current through the coil, and subsequently we lower the current to zero, and we cut the wire to remove the coil. To study the magnetic field of this ferromagnet, we make two of them and bring them close together. Explain the interaction between these two ferromagnets. Do they attract each other?

2. Consider two concentric circular loops, each carrying a steady current **I**, but in opposite directions.

The situation is indicated in the figure on the right. The two loops lie in the x-y plane. The radii of the loops are R_1 and R_2, with $R_1 < R_2$. The magnitude of the current is the same in both loops.



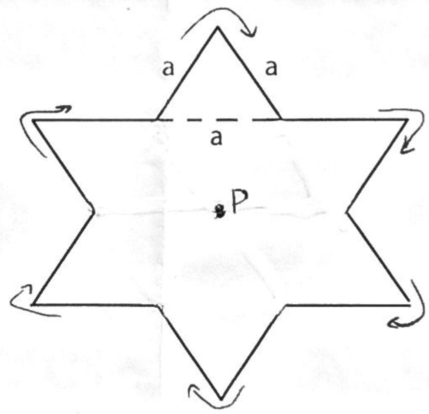
(a) (5 points) Use the Biot-Savart law to calculate the magnetic field **B** in the center of the loops ((x,y,z)=(0,0,0)).

(b) (5 points) Sketch the magnetic field lines in the (z,y) plane, for the situation where $R_1 = \frac{3}{4}R_2$, where the range for both y and z is $(-\frac{3}{2}R_2, \frac{3}{2}R_2)$. Make sure to indicate the direction of the magnetic field, and use the density of the field lines to indicate the strength (qualitatively) of the field. You do not need to do the calculation of the field for this subquestion.

3. (10 points) A living object like a small frog can be levitated, as has been demonstrated by Andrey Geim and his colleagues at the University of Nijmegen in 1997. Explain the mechanism responsible for the magnetism induced in a frog (on an atomic level), and make a sketch of the shape of the magnetic field required to levitate a frog, where you indicate all relevant forces.

4. A current loop

(a) (5 points) Consider the shape on the right. A current I flows through a wire that is bent in this star-shape, with the direction indicated by the red arrows. Find the magnetic field at point P due to the wire. Hint: consider that the star-shape can be created from two equilateral triangles, minus a hexagon in the center.



(b) (5 points) Find the magnetic dipole moment **m** of this current loop. Give the direction in which it points.